BLOO
Band-Limited Overlapping Oscillator

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Abstract

Study on bandlimited periodic signals and implications on sound-synthesis context. Generic synthesis algorithm is derived by imposing the concept of master oscillator frequency (in the hard-sync sense) to an existing signal generator which takes the part of the slave and may or may not have consistent tuning. Genericity, aliasing attenuation and restrictions are discussed.

1 Convolution-based representation of integer-periodic signals

The ideally bandlimited discrete-time periodic impulse train has the form [1]: [1]

$$iBLIT_P[n] = \sum_{k=-\infty}^{\infty} sinc(\pi(n - kP)) \tag{1}$$

where $P = T_p/T_s$ with T_p and T_s the fundamental and sampling periods respectively in continuous-time units, while $sinc(x)$ is the sampling function defined as [5]:

$$sinc(x) = \begin{cases} 1 & x = 0 \\ \sin(x)/x & \text{otherwise.} \end{cases} \tag{2}$$

It is easy to show that when T_p is an integer multiple of T_s, i.e.:

$$P = M \tag{3}$$

(1) reduces to:

$$iBLIT_M[n] = \sum_{k=-\infty}^{\infty} \delta[n - kM] \tag{4}$$

where $\delta[n]$ is the unit impulse sequence defined as:

$$\delta[n] = \begin{cases} 1 & n = 0 \\ 0 & |n| = 1, 2, 3... \end{cases} \tag{5}$$

[1] Without loss of generality phase is set to $iBLIT[0] = 1$ (i.e. there's an impulse peaking at $n = 0$).

It is possible to represent any periodic signal with a period of M samples as convolution of (4) with one cycle of the signal, i.e.:

$$p_M[n] = \sum_{l=-\infty}^{\infty} s_M[l] \sum_{k=-\infty}^{\infty} \delta[n - kM - l] \tag{6}$$

where p_M is the periodic and s_M the, causal, single cycle signal:

$$s_M[n] = r[n]u[n]u[M - 1 - n] \tag{7}$$

with u the unit step sequence defined as:

$$u[n] = \begin{cases} 1 & n \geq 0 \\ 0 & \text{otherwise.} \end{cases} \tag{8}$$

and r an infinite source of waveform data the nature of which characterizes many wavetable-based synthesis methods and techniques as it will be later discussed.

2 Generalizing for fractional periods

In (6) we describe a discrete-time periodic signal with a period of M samples in terms of convolution of an impulse train with the single cycle signal s_M. In the case where P is not an integer, we are in front of the need to define an analogous "fractionally"-long discrete-time single cycle signal in order to be able to deal with discontinuities introduced when advancing continuously between successive integer periods. We will represent this "fractionally"-long sequence in terms of linear interpolation between $s_{\lfloor P \rfloor}$ and $s_{\lceil P \rceil}$, i.e. between neighboring integer-period single-cycle signals of type s. Specifically:

$$s_P[n] = (1 - a)s_{\lfloor P \rfloor}[n] + as_{\lceil P \rceil}[n] \tag{9}$$

with

$$a = P - \lfloor P \rfloor \tag{10}$$

Now, generalizing (6) for fractional periods and in accordance to (9) and (1), an ideally bandlimited discrete-time periodic signal with a period of P samples has the form:

$$p_P[n] = \sum_{l=-\infty}^{\infty} s_P[l] \sum_{k=-\infty}^{\infty} sinc(\pi(n-kP-l)) \quad (11)$$

which after moving the inner sum to the left becomes:

$$p_P[n] = \sum_{k=-\infty}^{\infty} \sum_{l=-\infty}^{\infty} s_P[l] sinc(\pi(n-kP-l)) \quad (12)$$

reducing to (6) for integer P.

3 First observations

In the representation of (11), viewed from a linear filtering perspective, s_P can be seen as the kernel of a filtering process acting on the impulse train; it is the banlimited and harmonic nature of the impulse train's spectrum that ensures the same on the resulting signal through spectral multiplication caused by convolution [5]. Additionally, by examining (12), we see that the inner sum describes the convolution of the single-cycle signal s_P with a bandlimited impulse having a certain phase offset, or, in other words, s_P ideally delayed by a fractional number of samples. From there, the periodic signal is constructed by summing up the periodically delayed versions of s_P.

In the model suggested by (7), the integer-period single-cycle signal s_M is extracted by the waveform data r by means of causal rectangular windowing (the $u[n]u[M-1-n]$ term) and thus the value of M has no effect on r. Remaining under this assumption, s_M signals of successive Ms differ only by the presence or not of the last non-zero sample of the bigger M signal. Formally:

$$s_M[n] = s_{M+1}[n] - s_{M+1}[M]\delta[n-M] \quad (13)$$

Now (9) becomes:

$$s_P = s_{\lfloor P \rfloor}[n] + a s_{\lceil P \rceil}[\lceil P \rceil - 1]\delta[n - \lceil P \rceil + 1] \quad (14)$$

or, in a more logical and implementation-suggesting form:

$$s_P = \begin{cases} a s_{\lceil P \rceil}[n] & n = \lceil P \rceil - 1 \\ s_{\lceil P \rceil}[n] & \text{otherwise.} \end{cases} \quad (15)$$

From (15) we see that the "fractionally"-long signal can be constructed only from $s_{\lceil P \rceil}$ by weighing its last non-zero sample.

4 Time variance and the nature of r

As discussed above, the model assumed by (7) suggests that r does not change with T_p or time (in the sense that for any n, r has always the same value). Under this assumption, smoothly approximating "fractionally"-long single-cycle signals is done by linearly interpolating neighboring integer-long ones. Again from a linear filtering perspective, this transition between integer periods corresponds to a smooth spectral transition as well that happens to be totally phase-consistent in the sense that no unwanted phase cancellations can happen by mixing the two signals.

In practical applications, however, r tends to vary with respect to T_p and less often, time, something that, on the other hand, gives us the opportunity to characterize a bunch of wavetable-based synthesis approaches with fortunate implications regarding aliasing attenuation.

In the specific case where r is the output of a (phase-)resetable bandlimited wavetable oscillator of period T_{slave}, we have succesfully imposed a master frequency ($1/T_p$) to the oscillator, a concept inherent in hard-synchronization context ([3], [4]) and at the same time we have deterministic and bounded aliasing attenuation in an interesting form: the quality of the approximation of the fractionally delayed s_P determines, in essence, the ideality and thus harmonicity of the BLIT and consequently, that of the resulting signal. [2] However, any aliasing produced by the internal mechanisms of the slave oscillator that will also coincide with the BLIT harmonics, will get through. That means that for a precise arbitary-waveform wavetable synthesis with full hard-sync support both the overlapping wavetables (s_P) and the slave oscillator have to be sufficiently bandlimited. During the presence of aliasing in the slave's output, only the partials that coincide with the harmonics of the impulse train can make it through; the output after this method has a harmonic spectrum and thus increased musical potential.

Additionally, any time-varying behaviour of T_p and r is acceptable assuming it is sufficiently smooth; the first because abrupt and frequent changes in T_p introduce FM-induced aliasing and the second because, from a linear filtering perspective, rapid variations between successive periods correspond to rapid spectral variations as well, which multiply the impulse train's spectrum (also an impulse train), introducing aliasing by the individual amplitude modulation of harmonics. Both non-linear cases.

[2] The kernel s_P acts as a filter on the BLIT

5 More synthesis

By relaxing the constraint the single-cycle signal's length to be equal to P (with fractional lengths still supported by (15)), we can generalize for granular/formant-preserving synthesis with the same attenuation characteristics as traditional wavetable synthesis above. Because of the harmonic nature of the output, nonlinear synthesis methods like Phase Distortion, as any kind of harmonic FM/PM, may benefit significantly, even though the result might not be identical with that of a brute-force oversampled rendering; however, the resulting signal is guaranteed to be harmonic, a fact that is independent of the nature of r (assuming sufficiently smooth time variance) and thus the mechanisms producing it.

6 Computational efficiency

The generation of the fractionally delayed impulses in (12) goes down to lowpass FIR filter design techniques [2] [3] in conjuction with polyphase filtering ([3], [1]) since each discrete-time impulse corresponds to a specific phase of a lowpass filter kernel. However, issues like (some samples of) latency may arise depending on the method used for the filter design. Minimum phase conversions can minimize the amount of latency (more often to some fractional value) with respect to a specific magnitude response. It is possible to get acceptable quality for musical applications with phases of 8-samples long sub-kernels with nearest neighbor selection, or by using some higher order interpolation between phases to possibly reduce their number, length or both in memory-restricted enviroments.

In a sense, the method works as a higher level abstraction that actually imposes a master frequency to an oscillator with potentially some inherent tuning mechanism, by reseting it at periodic intervals and filtering the output accordingly. The length, in samples, of a slave oscillator's output after a reset determines the maximum number of overlapping oscillators at any time, along with the master period P [4]. Specifically:

$$O_{max} = \lceil \frac{\lceil W \rceil}{\lfloor P \rfloor} \rceil \tag{16}$$

where O_{max} is the maximum number of oscillators overlapping at any time, while W is the length of a slave oscillator's output after a reset (fractional length supported via (15)). In situations where $W \leq P$, we can see that the maximum overlapping oscillators operating at any time is 3. This also includes the important case where $W = P$, which was considered in the theoretical part of this article and contains what's most known as arbitary-waveform oscillator hard-sync as a subcase, with the literature dealing mainly only with "classic" waveforms of analog synthesizers or ones with sufficiently low C_n continuity ([3], [4]). For larger W the maximum number of overlaps could still be reduced, assuming no time-variance, by transforming $s_{\lceil W \rceil}$ to a time-aliased version with $\lceil P \rceil$ non-zero samples, since this is sufficient amount of information for a periodic signal of period P to be constructed.

7 Implementation

Following, is a pseudo-C++, object-oriented algorithmic formulation of the BLOO mechanism in a per-sample basis, for the case where $W = P$ (aka hard-sync). An oscillator, in this context, has a (phase) reset and a (per-sample) "tick" functions. Therefore, a BLOO interfaces like any other oscillator and can thus be a slave itself. Additionally, a circular buffer mechanism is utilized in order to precompute overlapping segments during a period "tick".

```
bloo::tick () {
  nPn -= 1;
  if (!floor(nPn)) {
    s = slave->tick();
    buf_N[idx] += aafilt->proc(s);
    s = nPn * slave->tick();
    tidx = add_wrap_N(idx+1);
    buf_N[tidx] += aafilt->proc(s);
    //loop can be opt. filter input 0,
    //add_wrap_N is independent
    for (i = 2; i <= N; i++) {
      tidx = add_wrap_N (idx+i);
      buf_N[tidx] += aafilt->proc(0);
    }
    //prob. also aafilt->zero_state();
    aafilt->set_phase(nPn);
    slave->reset();
    nPn += P;
  }
  s = slave->tick();
  s = aafilt->proc(s);
  s += buf_N[idx];
  idx = add_wrap_N(idx+1);
  return s;
}
```

[3] Allpass fractional delay approximations could also be used, however, because of their IIR nature, overlapping might be potentially more dense as truncating their impulse response significantly could mean lowering the quality.

[4] Normally, there's also the "tail" $N - 1$ of the polyphase sub-kernel to add in the length of W. However, that does not require any oscillator input; it operates on zero samples.

```
bloo::reset () {
  nPn = 0;
  aafilt->zero_state();
  slave->reset();
  zero(buf_N);
  idx = 0;
}
```

```
add_wrap_N (idx) {
  t = idx+1;
  if (t >= N) t -= N;
  //or t += max(t-N,0);
  return t;
}
```

8 Summary

We presented a method for deterministic, bounded and efficient attenuation of the inharmonic spectral components produced by an arbitary signal generator with a reset function, with respect to a master frequency imposed by the method, under the assumption that the master frequency doesn't change too abruptly and/or often and signal generator produces sufficiently similar spectra between resets. Arbitary-waveform oscillator hard-sync is covered as a subcase.

References

[1] Tim Stilson, Julius Smith *Alias-Free Digital Synthesis Of Classic Analog Waveforms* ICMC 1996.

[2] V. Välimäki, T. I. Laakso *Principles Of Fractional Delay Filters* ICAASP 2000.

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[4] Eli Brandt *Hard Sync Without Aliasing* ICMC 2001.

[5] Alan V. Oppenheim, Ronald W. Schafer, John R. Buck *Discrete-Time Signal Processing (2nd Edition)* Prentice Hall, 1999.